# Mutual Coupling Between Waveguide Apertures Mounted on a Common Conducting Surface Using a Time- and Fourier-Gated Pulsed FDTD Method

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*Abstract*— **A time and Fourier gated pulsed finite difference time domain (FDTD) algorithm for computing mutual coupling between waveguide apertures is formulated. The method uses an incident pulse of short time duration (wide bandwidth) and a wide band ABC for lossless waveguides. It is capable of the computation of scattered or mutually coupled fields even if the intensity of the required fields is lower than the inherent reflection levels of the free space artificial truncation planes. The method is illustrated by computing the mutual coupling between two rectangular waveguides mounted on a common groundplane and bounded by a parallel plate waveguide.**

## I. Introduction

In this letter, the finite difference time domain (FDTD) analysis of mutual coupling between rectangular waveguide apertures mounted on a common conducting surface is considered. The surface on which the apertures are mounted and the apertures themselfs are assumed to conform to a rectangular coordinate system. In the design of aperture arrays, and in the positioning of antenna apertures, a knowledge of the mutual coupling between the individual apertures is essential.

The analysis of mutual coupling problems has been undertaken for a number of problems frequently encountered in practice. Mutual coupling between apertures on a common ground plane, on a cone and on a spherical surface is presented in [1]–[3]. A. Peterson and R. Mittra [4] proposed a numerical technique for the determination of the mutual coupling between waveguide apertures mounted on an infinite two dimensional cylinder of arbitrary cross section.

In this letter, the mutual coupling problem is solved numerically via a direct solution of the differential form of Maxwell's equations subject to the appropriate three dimensional boundary conditions, using the FDTD method [5]–[9]. The formulation will not assume only a dominant $TE_{10}$ mode at the aperture as in [4], but will include the effects of the generation of evanescent higher order modes. The FDTD method directly solves the differential form of Maxwell's equations in a volumetric computational domain. The computational domain is truncated artificially via a suitable Absorbing Boundary Condition (ABC) [10]–[17].

Manuscript received February 8, 1993.
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IEEE Log Number 9209672.

In Section II, the time and Fourier gated FDTD algorithm is formulated. Suitable equations for the incident field are also given. Section III gives a numerical example and Section IV concludes the letter.

## II. The Pulsed Incident Field and Wide-Band Absorbing Boundary Condition

Consider two rectangular waveguide fed apertures mounted on a common conducting surface. For an incident field in a rectangular waveguide we consider a pulse modulated $TE_{10}$ mode given by

$$\boldsymbol{E}(x,y,z,t) = \phi(x,y)f(z,t)e_y = \sin\left(\frac{\pi x}{a}\right)f(z,t)e_y, \quad (1)$$

where $x$ and $y$ are the transverse coordinates, $z$ the axial coordinate and $t$ the time. The waveguide has a wide dimension $a$, and narrow dimension $b$. We define $f(z,t)$ in terms of $f(z=0,t)$ while the latter function is defined in terms of its Fourier transform. Hence, $f$ is given by

$$f(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(z=0,\omega)e^{j(\pm\beta(\omega)z-\omega t)}d\omega, \quad (2)$$

where $\beta$[1] is the $TE_{10}$ mode propagation constant given by

$$\beta(\omega) = \frac{\omega}{c}\sqrt{1-\left(\frac{c\pi}{\omega a}\right)^2}, \quad (3)$$

with $c$ the speed of light. Equation (2) is usually not amenable to a closed form solution, and is therefore integrated numerically. Inside the rectangular waveguides at a distance $z_{ABC}$ from the apertures we place the artificial truncation plane where we enforce the wide band waveguide ABC. The waveguide ABC is given by [8], [11],

$$\frac{\partial^2}{\partial z\partial t}\psi(z,t) \pm \frac{1}{c}\frac{\partial^2}{\partial t^2}\psi(z,t) \pm \frac{1}{2c}\left[\frac{c\pi}{a}\right]^2\psi(z,t) \approx 0. \quad (4)$$

A finite region of the surrounding free space is selected and artificially truncated using a free space ABC [10]–[12]. The sum of the portions inside the feed waveguides and the selected part of free space is referred to as the volumetric computational domain $\mathcal{E}$. The domain $\mathcal{E}$ is discretized or filled with a finite

[1]For positive/negative propagating waves, the negative/positive sign for $\beta$ is used.

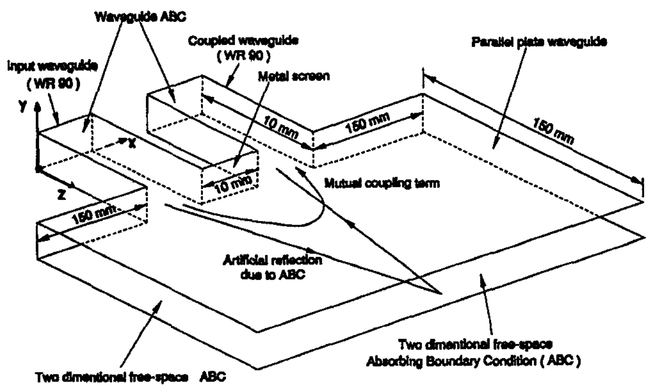


Fig. 1. A two-dimensional mutual coupling problem as an illustration of the technique.

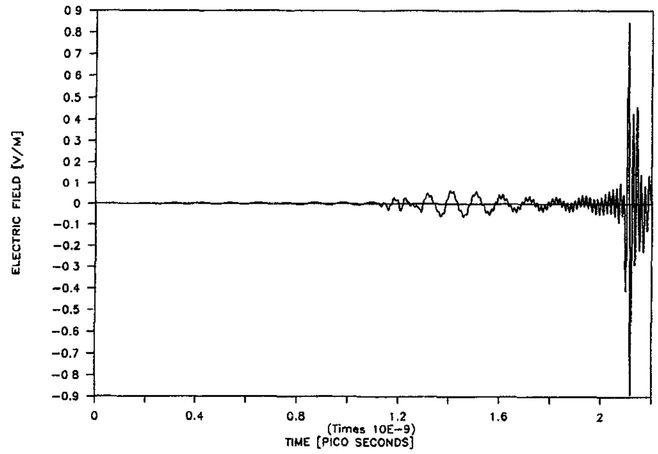


Fig. 2. The time domain response of the physical setup shown in Fig. 1.

number of Yee unit cells[2]. Suitable boundary conditions are enforced at the conducting boundaries. The FDTD method is now utilized to simulate the propagation of the incident pulse in the domain $\mathcal{E}$.

## III. Formulation of the Gated FDTD Algorithm

In this section, we will outline the procedure and compare measured versus predicted results for the setup shown in Fig. 1. The waveguides in Fig. 1 operate in the X-band frequency range with a cutoff frequency of 6.65 GHz. The incident pulse may therefore theoretically contain any frequency component above 6.65 GHz, and in this case, we use an incident pulse with a flat spectrum between 9 and 69 GHz. This will ensure that the mutual coupling term and free space ABC reflected terms be separated in time. The dimensions of the computational domain are indicated in Fig. 1. The X-band waveguides have dimensions of 22.86 × 10.16 mm, and the waveguide ABC's are placed 10 mm from the waveguide apertures. This is sufficient for the evanescent modes generated at the aperture plane to have decayed to negligible values at the waveguide ABC planes (propagating higher order modes will be gated out on the frequency domain). The incident pulse is also retarded in time so that the pulse has a negligible value at time $t = 0$. This is done so that the so called smooth turn-on condition is satisfied. In this particular case, the pulse is retarded by 1000 ps. A first-order free-space ABC [11] was used to terminate the mesh in the two-dimensional free-space region. The use of a free-space ABC is strictly speaking not essential since reflections from the free-space ABC is gated out. Its use will however indicate that the ABC reflected term is large compared to the sought after term, indicative of the necessity of the current formulation. Note that the dominant mode reflected fields at the waveguide ABC's are not gated out as they are orders of magnitude smaller than the fields at the waveguide ABC termination planes.

The reflected pulse amplitude due to the free space ABC's may be higher than the mutual coupling terms due to the small value of the mutual coupling term. Under such circumstances the usual FDTD algorithm will lead to extremely inaccurate results. This problem may be circumvented by isolating the terms in time, and then "gating out" the sought after term. Time isolation is possible by increasing the size of the portion of the discretized part of free space, and/or decreasing the time duration of the incident pulse.

The use of a short pulse generates propagating high-frequency, higher order modes inside the feed and receiving waveguides. These may be gated (filtered) out on the frequency domain. This is easily accomplished by computing the FFT [18] of the time domain fields inside the "time gate" at the receiving ABC plane, but only at frequencies below the waveguide cutoff frequency. Hence, the transmission coefficient between waveguide 1 and 2, i.e., $S_{21}$ is automatically computed at all frequencies in the dominant mode range of the waveguide in a single application of the procedure. Fig. 2 show the predicted time response of the transmission coefficient $S_{21}$ with reference to the ABC truncation planes in the feed and receiving waveguide. The Yee cell size is kept at 0.5 mm$^2$ throughout, which is sufficient to resolve the smallest wavelength and physical dimension. The mutual coupling term is evident after approximately 1100 ps. Note the "ringing" at later times in Fig. 2. This is attributed to the presence of propagating high frequency higher order modes. When the time has increased to 2100 ps the pulse reflected from the free space ABC arrives. Note its large amplitude relative to the mutual coupling term.

The time gate should therefore start at approximately 1100 ps, and end before the arrival of the ABC reflected term, i.e., before 2000 ps. Fig. 3 shows the Fourier transformation (of $S_{21}$) of the pulse present inside this time gate. The Fourier transformation of $S_{21}$ is computed at frequencies permitting the propagation of the dominant $TE_{10}$ mode only, i.e., between 9 and 12.4 GHz. Hence, the propagating (high-frequency) higher order modes are gated (filtered) out on the frequency domain. Measured values for the transmission coefficient $S_{21}$ are also shown in Fig. 3. The correspondence is fair except at the high-frequency end where a deviation of approximately 4 dB is observed. The difference between the predicted and measured values is attributed to measurement error, due to difficulties that were encountered in isolating such a small signal in the time domain using a network analyzer.[3]

[2] A basic familiarity with the FDTD algorithm and Yee cell is assumed.

[3] The phase of $S_{21}$ were computed but the measured phase were found not to be reliable. Hence, it is not included in this letter.

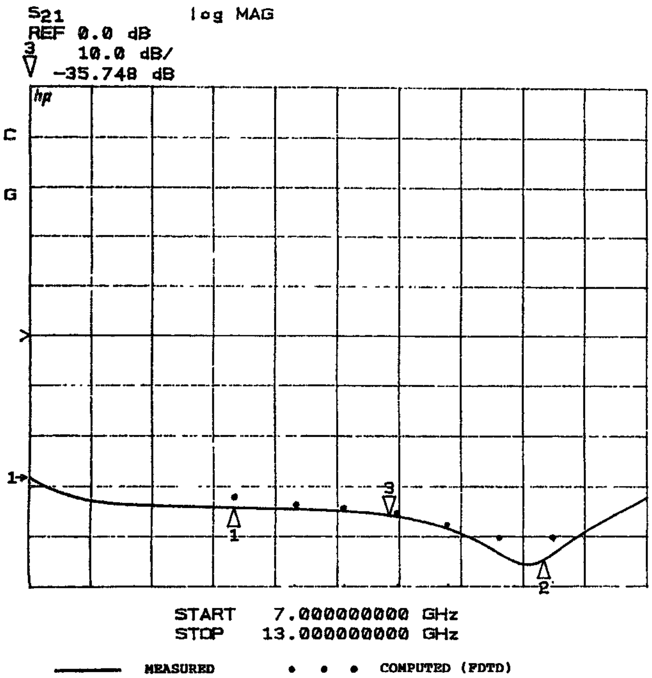


Fig. 3. Transmission coefficient or Fourier transform of the time-domain response (Fig. 2) inside the time gate extending from 1100 ps to 2000 ps.

## IV. Conclusion

A time- and Fourier-gated pulsed FDTD method was formulated for the numerical determination of the mutual coupling between waveguide apertures mounted on a common surface. The method was illustrated by applying it to a representative two dimensional application. Measurements agree well with computed values for the mutual coupling.

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